A Decomposition of the Teaching Practice of Building

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Abstract

We share a decomposition of building on MOSTs—*a teaching practice that takes advantage of high-leverage instances of student mathematical contributions made during whole-class interaction. This decomposition resulted from an iterative process of teacher-researchers enacting conceptions of the building teaching practice that were refined based on our study of their enactments. We elaborate the four elements of building: (a)* Establish *the student mathematics of the MOST as the object to be discussed; (b)* Grapple Toss *that object in a way that positions the class to make sense of it; (c)* Conduct *a whole-class discussion that supports the students in making sense of the student mathematics of the MOST; and (d)* Make Explicit *the important mathematical idea from the discussion. We argue for the value of this practice in improving in-the-moment use of high-leverage student mathematical thinking during instruction.*

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One hallmark of effective mathematics instruction is intentional and pervasive use of student mathematical thinking (NCTM, 2014). Researchers have argued, however, that such use is still not well understood (e.g., Robertson et al., 2016). Furthermore, it has been posited that some instances of student thinking are of more mathematical importance than others (e.g., Choy, 2014) and that using them productively can be especially advantageous to student learning (Thames & Ball, 2013). Taking full advantage of such instances—instances we have referred to as **M**athematical **O**pportunities in **S**tudent **T**hinking or MOSTs (Leatham et al., 2015)—requires the coordination of a collection of teaching actions into a complex practice, including some actions that do not occur naturally in whole-class instruction (Stockero et al., 2020). To better understand and improve teachers' ability to engage in complex practices, Grossman and colleagues (2009) suggested that practices be decomposed into their "constituent parts" (p. 2069). In this paper we share a decomposition of a teaching practice that takes full advantage of MOSTs, thus contributing to a better understanding of productive use of student mathematical thinking.

MOSTs occur at the intersection of three critical characteristics of classroom instances: student mathematical thinking, significant mathematics, and pedagogical opportunity. In essence, MOSTs are student mathematical contributions that provide an in-the-moment opportunity to engage students in joint sense making. *Building on MOSTs* is a teaching practice that takes advantage of the opportunity that a MOST provides (Leatham et al., 2021) in a way that adheres to what we call *Core Principles* of effective teaching of mathematics: students are positioned as legitimate mathematical thinkers, student mathematics is at the forefront, students are engaged in sense making, and students are working collaboratively (NCTM, 2014; Stockero et al., 2020). We formally define *building on MOSTs* (hereafter referred to as *building*) as *engaging the class in making sense of a MOST to better understand the mathematics of the MOST*. As described elsewhere (Leatham et al., 2021), unpacking that definition in the context of our collective experience with analyzing teaching (our own and that of others) and through the lens of the *Core Principles* led us to theorize building as being comprised of four elements (Figure 1): (a) *Establish* the student mathematics of the MOST as the object to be discussed; (b) *Grapple Toss* that object in a way that positions the class to make sense of it; (c) *Conduct* a whole-class discussion that supports the students in making sense of the student mathematics of the MOST; and (d) *Make Explicit* the important mathematical idea from the discussion.

Figure 1. The four elements of building on MOSTs.

Methodology

We engaged in an iterative process around 12 secondary teacher-researchers' successive classroom enactments of our evolving conceptions of building. We coded these videorecorded, transcribed enactments according to what each teacher action seemed to accomplish with respect to building. We documented why particular teacher actions seemed to either facilitate or hinder the overall practice of building as measured against our *Core Principles*. We present here our refined conception of building, including key aspects of each element and a variety of associated subtleties.

Results

Establish

As shown in Figure 2, the four aspects of the Establish element are to establish (a) precision, (b) an object, (c) intellectual need, and (d) a conversational bubble. In the following sections we elaborate on these aspects and associated teacher actions (see Leatham et al., 2021 for additional details).

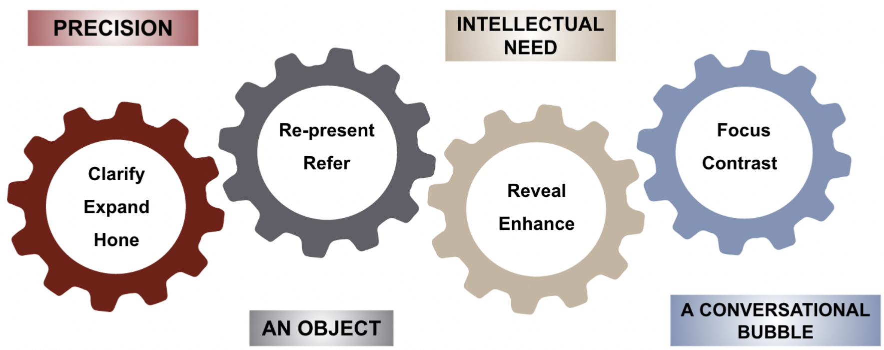

Figure 2. The Establish element of building on MOSTs.

Precision. The first aspect of the Establish element involves the teacher ensuring that the MOST is clear, complete, and concise, which may require the teacher actions of clarify, expand, and hone. Clarification makes the contribution itself clear and is required when student contributions contain vague referents or when the logical structure of their ideas is not transparent. Expansion makes the contribution complete and is required when student contributions provide answers without reasoning (most common) or when necessary information is omitted. Honing makes the MOST concise when (a) the student contribution contains unnecessary verbiage, or (b) ideas within the contribution can be more succinctly captured by using symbols or other shorthand (typically on the board).

An object. In addition to establishing precision, the Establish element also entails the work of ensuring that the MOST is established as an object, as a "thing" that can be considered by everyone in the class. This aspect involves the teacher actions of re-present and refer. Re-presenting the MOST through repeating, revoicing, or creating a public record of that contribution creates object permanence. Referring to those re-presentations through clear referents, gestures, or naming the contribution (e.g., this claim, Tray's thinking) helps to create object identity.

Intellectual need. Beyond establishing the MOST as a precise object, the third aspect involves establishing the intellectual need students might have to make sense of the contribution. One critical characteristic of a MOST is that it presents a pedagogical opportunity; one criterion for a pedagogical opportunity is that there be some evidence that making sense of the contribution will help to satisfy an intellectual need (Leatham et al., 2015). This aspect involves the teacher actions of reveal and enhance. When the intellectual need is hidden (such as when everyone thinks an incorrect answer is correct), the teacher needs to reveal that need (e.g., "This answer you all agree on is wrong.") in order to prepare the class to make sense of the contribution. The intellectual need of a MOST can be enhanced by establishing the commonality of the thinking (e.g., "I heard a number of students express this same idea.") or by establishing the existence of a contradiction (e.g., "I can see that we do not all agree.").

A conversational bubble. The fourth and final aspect entails establishing a conversational bubble in which the pursuant discussion about the MOST can take place. Because MOSTs often occur during ongoing whole-class interactions, students need explicit help recognizing the shift in activity that building on that MOST represents. Teachers create a conversational bubble by inviting explicit focus on the MOST (e.g., "Let's take a minute to consider this claim."). At times they might create this focus by contrasting the building activity with the activity that had been occurring (e.g., "Rather than share your own solution, let's just consider this solution for a moment.")

Grapple Toss

Having established the MOST as a precise object that has intellectual need, the teacher then offers this established object to the class to make sense of. There are two aspects of this Grapple Toss: the object and the sense-making action. With respect to the object, optimally the teacher takes advantage of the prior Establish work (e.g., by referring to the established MOST by name). As seen in Figure 3, the specific sense-making action of a Grapple Toss question depends on the nature of the MOST—a *question*, a *claim or solution*, *revealed* intellectual need, or *multiple claims or solutions*. An effective Grapple Toss question is as specific as possible while maintaining the cognitive demand (Stein et al., 1996) of the sense-making activity.

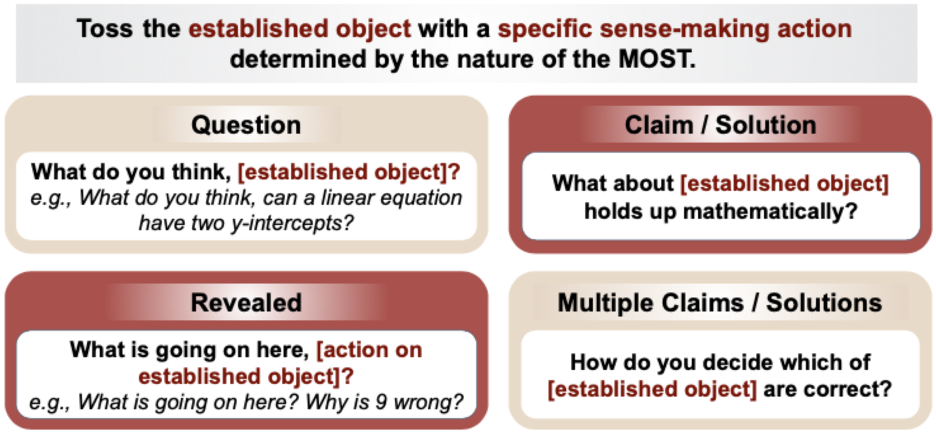

Figure 3. The Grapple Toss element of building on MOSTs.

Conduct

Once the MOST has been Established as a precise object that has intellectual need and has been Grapple Tossed to the class, the teacher conducts a whole-class discussion that supports the students in making sense of the MOST. The flowchart in Figure 4 illustrates the iterative nature of the Conduct element of building. Beginning with the initial student response to the Grapple Toss, the first aspect of Conduct is determining whether a student contribution is a *MOST-Related Contribution* (MRC)—*a student contribution that has the potential to help the class make sense of the MOST.* What happens next depends on whether the contribution is an MRC.

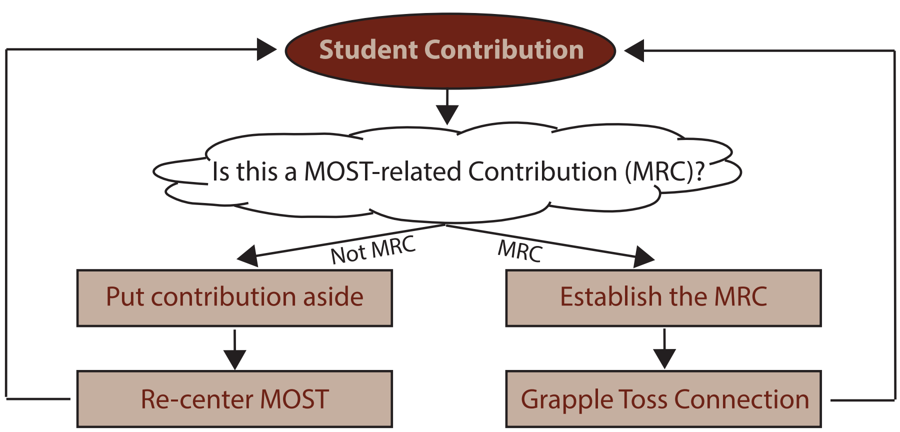

Figure 4. The iterative process of the Conduct element of building on MOSTs.

Response to a non-MRC. Since student contributions that are not MRCs will distract from rather than contribute to the discussion, the most effective response is to put a non-MRC aside and re-center the MOST (e.g., “Remember, we are making sense of Jaden’s claim right now.”). If the non-MRC has diffused the momentum of making sense of the MOST, it may be necessary to re-Grapple Toss the MOST to reposition students to make sense of it.

Response to an MRC. When a student contribution is a MOST, the goal is to explicitly ask students to use this new information (the MRC) to engage in making sense of the MOST—to make connections between the two ideas. In essence, variations of the Establish and Grapple Toss elements play out when one responds to an MRC. First, the MRC needs to be Established so that the students know what will be connected to the already established MOST, thus ensuring that the two ends of the connection have been Established. Second, the connection between the MRC and the MOST needs to be Grapple Tossed to the class (what we refer to as Grapple Toss Connection). In the same way that the Grapple Toss of an established MOST tells the class the action they are to take on the MOST, the Grapple Toss Connection tells students the action they are to take on the MRC and the MOST—to use the MRC to help make sense of the MOST (e.g., How does Shea’s claim help us better understand what is going on with Jaden’s claim?). A student response to the Grapple Toss Connection recommences the iterative process of the Conduct element, as a new student contribution is generated that may or may not be an MRC.

Make Explicit

The final element of building entails ensuring that the important mathematical ideas from the sense-making discussion (the mathematical takeaways) are made explicit. There are three aspects of the Make Explicit element: Resolution, Generalization, and Transition (see Figure 5). Each aspect and related teacher actions are discussed below.

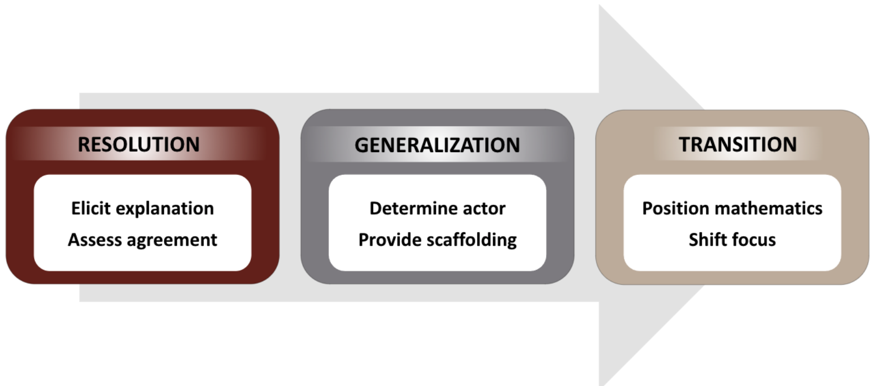

Figure 5. The Make Explicit element of building on MOSTs.

Resolution. The first aspect of Make Explicit is ensuring that students have made sense of the MOST (that the MOST has been resolved), in essence confirming that the Conduct element has indeed concluded. A typical action to elicit this resolution is to check in with the class by re-Grapple Tossing the MOST. If students have made sense of the MOST, this re-toss often results in a relatively succinct answer to the Grapple Toss question. At times such an explanation emerges organically as the Conduct element concludes; in this case teachers can acknowledge the resolution as being such. With a resolution on the table, the teacher assesses whether there is collective agreement with this resolution, sometimes through observation of students' reactions, other times through posing a question like, "Do we all agree with this statement?" (If attempts to resolve the MOST are not successful, then the Conduct element has not concluded and further Conduct actions are warranted.)

Generalization. Beyond making explicit the resolution of the MOST itself, to truly take advantage of the MOST the teacher ensures that this resolution is connected to the broader mathematical point afforded by the MOST. In other words, whereas the resolution of the MOST is local—specific to the issue at hand (e.g., $x+x$ isn't necessarily bigger than x because x isn't necessarily positive), the mathematical point afforded by the MOST is a generalization beyond the specifics of the MOST itself (e.g., The domain of the variable must be considered to determine the relative values of variable expressions). The teacher determines who makes this generalization (the actor), considering whether students can be prompted to provide a generalized mathematical statement or they themself need to articulate the generalization. Regardless of the actor, it is important for teachers to provide appropriate scaffolding for this generalizing work (e.g., use or introduce formal mathematical language).

Transition. Finally, because (a) MOSTs occur in the midst of ongoing whole class discussion, and (b) the significant mathematics of a MOST is not always central to the mathematics of the lesson in which it occurs, teachers need to make explicit the relative importance to the lesson of this generalized mathematical idea and then move on. At times such a transition marks a return to what was happening before (e.g., "I'm so glad we had the chance to better understand this mathematics that is such a critical idea in this class. Now, back to what we were doing."). At other times such a transition might build on the mathematics of the MOST (e.g., "So how can we use this mathematical idea to solve the problem we were working on?").

Discussion and Conclusion

We have elaborated on the four elements of building on a MOST: (a) *Establish* the student mathematics of the MOST as the object to be discussed; (b) *Grapple Toss* that object in a way that positions the class to make sense of it; (c) *Conduct* a whole-class discussion that supports the students in making sense of the student mathematics of the MOST; and (d) *Make Explicit* the important mathematical idea from the discussion. Each element sets the stage for those that follow. For example, the object that is established in Establish (the established MOST) serves as a focal point during each of the remaining three elements. During Grapple Toss the established MOST is what gets tossed; during Conduct the established MOST is one end of the connection with an MRC and is what is returned to after a non-MRC is put aside; and during Make Explicit the established MOST is the object of the resolution. We also see some self-similarity within the elements, as versions of Establish and Grapple Toss show up in the midst of Conduct. Thus, decomposing building into its "constituent parts" (Grossman et al., 2009, p. 2069) has helped us to better understand both the individual elements and how they work together.

Focusing on the teacher actions that make up building may make the practice seem teacher-centered. Although building is a teaching practice (defined by teachers' actions), it is not a teacher-centered practice. Quite to the contrary, the whole purpose of building is to make student mathematical thinking the center of instruction. For example, in Grapple Toss, the teacher actively positions the students to make sense of the established MOST—an instance of student mathematical thinking—by asking a question that captures the action they want the students to carry out in that sense-making (as captured in Figure 3). We have found that effective student-centered instruction requires substantial and intentional teacher work—mathematical work for teaching that positions the students to do mathematical work for learning.

Building has the potential to improve in-the-moment use of student mathematical thinking during instruction by codifying our *Core Principles* (mentioned previously). As evidenced by the teacher-researchers who have enacted the practice in their classrooms, being conscious of the different elements of building and of the teacher actions that comprise them enhances teachers' in-the-moment use of student mathematical thinking. Decomposing practices in the way we have done is an important first step in supporting teachers to develop these practices (Grossman et al., 2009).

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References

Choy, B. H. (2014). Noticing critical incidents in a mathematics classroom. In J. Anderson, M. Vananagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice (Proceedings of the 37th annual conference of MERGA)* (pp. 143–150). MERGA.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record, 111*(9), 2055-2100.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education, 46*(1), 88-124. https://doi.org/10.5951/jresematheduc.46.1.0088

Leatham, K. R., Van Zoest, L. R., Freeburn, B., Peterson, B. E., & Stockero, S. L. (2021). Establishing student mathematical thinking as an object of class discussion. In D. Olanoff, K. Johnson, & S. Spitzer (Eds.), *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1392-1400). PME-NA.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Author.

Robertson, A. D., Scherr, R. E., & Hammer, D. (2016). *Responsive teaching in science and mathematics*. Routledge.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488. https://doi.org/10.3102/00028312033002455

Stockero, S. L., Leatham, K. R., Ochieng, M. A., Van Zoest, L. R., & Peterson, B. E. (2020). Teachers' orientations toward using student mathematical thinking as a resource during whole-class discussion. *Journal of Mathematics Teacher Education*, *23*(3), 237-267. https://doi.org/10.1007/s10857-018-09421-0

Thames, M. H., & Ball, D. L. (2013). Making progress in U.S. mathematics education: Lessons learned—past, present and future. In K. Leatham (Ed.), *Vital directions for mathematics education research* (pp. 15–44). Springer.